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Inclusive hadronic distributions at small x inside jets

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Abstract: After giving their general expressions valid at all x, double differential 1-particle inclusive distribution inside a quark and a gluon jet produced in a hard process, together with the inclusive $k_\perp$ distribution, are given at small x in the Modified Leading Logarithmic Approximation (MLLA), as functions of the transverse momentum $k_\perp$ of the outgoing hadron.

Introduction

This work concerns the production of two hadrons inside a high energy jet (quark or gluon); they hadronize out of two partons at the end of a cascading process that we calculate in pQCD[1]. Considering this transition as a "soft" process is the essence of the "Local Parton Hadron Duality" (LPHD) hypothesis[2]. We give indeed, in the MLLA scheme of resummation, the double differential inclusive 1-particle distribution and the inclusive $k_\perp$ distribution as functions of the transverse momentum of the emitted hadrons in the limit that proved successful when describing energy spectra of particles in jets, that is $Q_0 \approx \Lambda_{QCD}$, the so called "limiting spectrum"[2].

The process under consideration

In a hard collision (pp, $p\bar{p}$ collisions, e^+e^- annihilation) we consider a jet of half opening angle Θ_0 initiated by a parton A_0, which can be a quark or a gluon, see Fig.1. A_0, by a succession of partonic emissions (quarks, gluons), produces a jet of half opening angle Θ_0, which, in particular, contains the parton A; A splits into B and C, which hadronize respectively into the two hadrons h_1 and h_2. Θ is the angle between B and C. Because the virtualities of B and C are much smaller than that of A, Θ can be considered to be close to the angle between h_1 and h_2; angular ordering (AO) is also a necessary condition for this property to hold. A_0 carries the energy E and gives rise to the (virtual) parton A, which carries the fraction u of the energy E. $A \rightarrow B + C$ occurs with probability $\propto \Phi$, Φ is the corresponding DGLAP splitting function, B and C carry respectively the fractions uz and $u(1-z)$ of E; h_1 carries the fraction x_1 of E; h_2 carries the fraction x_2 of E. One sets $\Theta \le \Theta_0$.

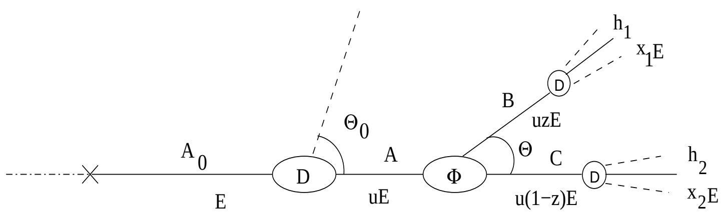

Fig. 1: process under consideration: two hadrons h_1 and h_2 inside one jet.

Since we are concerned with observables depending on the transverse momentum of outgoing hadrons,

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$k_\perp$ should be defined with respect to the jet axis which is identified with the direction of the energy flow[1].

Double differential 1-particle inclusive distribution at all values of x_1

The energy conservation sum rule, considered together with DGLAP evolution equations and AO lead the general expression for the double differential 1-particle inclusive distribution

$$\frac{d^2N}{dx_1\,d\ln\Theta}=\frac{d}{d\ln\Theta}\sum_A\int_{x_1}^1 duD_{A_0}^A\left(u,E\Theta_0,uE\Theta\right)D_A^{h_1}\left(\frac{x_1}{u},uE\Theta,Q_0\right) \tag{1}$$

which is valid at all x_1. $\frac{d^2N}{dx_1\,d\ln\Theta}$ in (1) is described indeed by the convolution of two fragmentation functions, $D_{A_0}^A$ and $D_A^{h_1}$, over the energy fraction u of A. Namely, $D_{A_0}^A$ is purely partonic and describes the probability to emit parton A with energy uE off parton A_0, moreover, it takes into account the evolution of the jet between Θ_0 and Θ; $D_A^{h_1}$ describes the probability to produce hadron h_1 of energy fraction x_1/u at angle Θ with respect to the direction of the energy flow inside the subjet A. On the other hand, the transverse momentum of h_1 should be bigger than the collinear cut-off parameter Q_0 ($uE\Theta\geq Q_0$). Consequently, a lower bound for Θ is obtained: $\Theta\geq\Theta_{min}\approx Q_0/x_1E$.

Perturbative expansion parameter and kinematics

We conveniently define the variables $\ell_1=\ln\frac{1}{x_1}$, $y_1=\ln\frac{x_1E\Theta}{Q_0}$. In what follows, we use the anomalous dimension γ_0 defined through the running coupling constant α_s as

$$\gamma_0^2(k_\perp^2)=\frac{2N_c\alpha_s(k_\perp^2)}{\pi}=\frac{2}{\beta\ln\dfrac{k_\perp^2}{\Lambda_{QCD}^2}}\equiv\gamma_0^2(\ell_1+y_1)=\frac{1}{\beta(\ell_1+y_1+\lambda)}=\frac{1}{\beta(Y_\Theta+\lambda)},\qquad\lambda=\ln\frac{Q_0}{\Lambda_{QCD}}. \tag{2}$$

It determines the rate of multiplicity growth with energy. $N_c=3$ for $SU(3)$, $\beta=\frac{1}{4N_c}\left(\frac{11}{3}N_c-\frac{4}{3}T_R\right)\overset{n_f=3}{=}0.75$ with $T_R=\frac{1}{2}n_f$ where $n_f=3$ is the number of light quarks, Λ_{QCD} is the QCD scale, $Y_\Theta=\ell_1+y_1=\ln\frac{E\Theta}{Q_0}$ and $Y_{\Theta_0}=\ln\frac{E\Theta_0}{Q_0}$. For instance, in the LHC environment we can take the typical value $Y_{\Theta_0}=7.5$ $\Rightarrow\gamma_0\simeq0.4$ by setting $\lambda=0$ ($Q_0=\Lambda_{QCD}$), $\Theta=\Theta_0$ and $Q_0=250$ MeV in (2). γ_0 can be therefore treated as the small parameter of the perturbative expansion at MLLA.

Double differential 1-particle inclusive distribution at small x_1, $x_1\ll1$

The convolution integral (1) is dominated by $u\approx1$. In order to obtain an analytical expression of (1), since $x_1/u\ll1$, we perform a perturbative expansion in γ_0 such that (1) gets factorized and derived in the region of soft multi-particle production[1]

$$\frac{d^2N}{d\ell_1\,d\ln k_\perp}=\frac{<C>_{q,g}}{N_c}\frac{d}{dy_1}\tilde{D}_g(\ell_1,y_1)+\frac{1}{N_c}\tilde{D}_g(\ell_1,y_1)\frac{d}{dy_1}<C>_{q,g}, \tag{3}$$

$$\frac{d}{dy_1}\tilde{D}_g(\ell_1,y_1)=\mathcal{O}(\gamma_0)=\mathcal{O}(\sqrt{\alpha_s}),\qquad\frac{d}{dy_1}<C>_{q,g}=\mathcal{O}(\gamma_0^2)=\mathcal{O}(\alpha_s). \tag{4}$$

The first term in (3) is the main contribution to the double differential 1-particle inclusive distribution while the second one constitutes its MLLA correction of relative order $\mathcal{O}(\gamma_0)$ (4). $D_A^{h_1}$ in (1) has been replaced by the distribution $D_g(\ell_1,y_1)$ in (3) that describes the MLLA "hump-backed plateau" in the limit where Q_0 can be taken down to Λ_{QCD} ("limiting spectrum")[2]. $<C>_{q,g}$ is the total colour current that describes the evolution of the jet between Θ_0 and Θ. It decomposes into its leading term $<C>_{q,g}^0$ and the MLLA correction $\delta<C>_{q,g}=\mathcal{O}(\gamma_0)$[1]. In Fig.2 we represent $<C>_{q,g}^0$ (straight line) and the full expression $<C>_{q,g}=<C>_{q,g}^0+\delta<C>_{q,g}$ (curve) for two values of ℓ_1; $\ell_1=$

2.5(left), 3.5(right) in function of y_1. Two types of MLLA corrections are displayed in this figure. Namely, $\delta < C >_{q,g} < 0$ is given by the vertical difference between the straight and curved lines. The other correction is given by the slope of the curve, that is $\frac{d}{dy_1} < C >_{q,g}$ in (4), this one is large and positive for $y_1 \geq 1.5$. For $\ell_1 = 2.5$ and $y_1 \approx 1.5$, $\delta < C >_{q,g}$ represents 50% of $< C >_{q,g}$ while for $\ell_1 = 3.5 > 2.5$ it gets under control. We are thus allowed to set the range of applicability of our soft approximation to $\ell_1 \geq \ell_{1,min} \approx 2.5$ ($x_1 \lesssim 0.08$) $\Rightarrow y_1 \leq y_{1,max} = Y_{\Theta_0} - \ell_{1,min} = 5.0$.

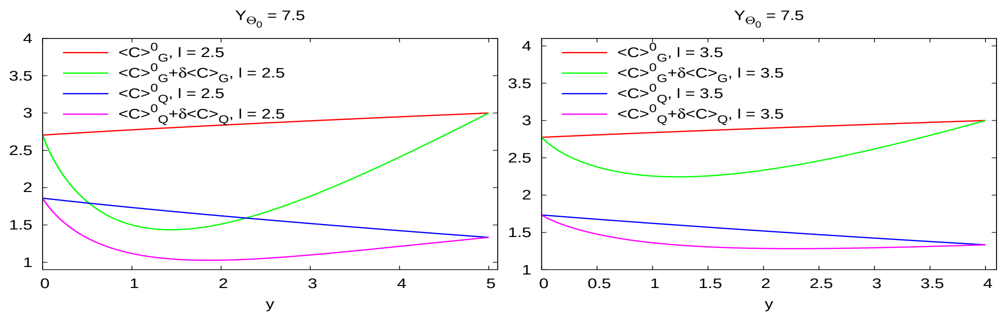

Fig. 2: $< C >^0_{A_0}$ and $< C >^0_{A_0} + \delta < C >_{A_0}$ for quark and gluon jets, as functions of y, for $Y_{\Theta_0} = 7.5$, $\ell = 2.5$ on the left and $\ell = 3.5$ on the right.

Furthermore, one should stay in the perturbative regime, which needs $y_1 \geq 1$ ($k_\perp > 2.72\Lambda_{QCD} \approx 0.7$ GeV). We finally get an estimate of the range of applicability of the MLLA scheme of resummation to be $1.0 \leq y_1 \leq 5.0$ in the LHC environment. In Fig.3 below, we represent the double differential 1-particle inclusive distribution (3) for $\ell_1 = 3.5$ in function of y_1. We compare our results with a naive DLA-inspired case where one does not take into account the evolution of the jet between Θ_0 and Θ but sets $< C >_q = C_F$ (quark jet) and $< C >_g = N_c$ (gluon jet).

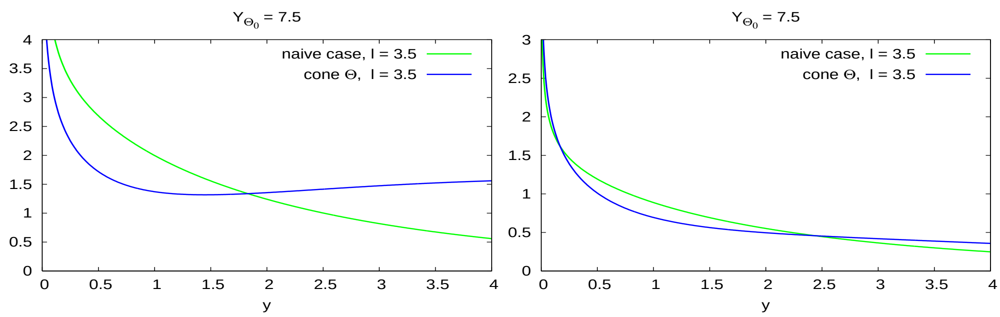

Fig. 3: $\frac{d^2N}{d\ell_1\, d\ln k_\perp}$ for a gluon jet (left) and for a quark jet (right) at fixed $\ell_1 = 3.5$, MLLA and naive approach.

For both, the quark and gluon jets, new MLLA corrections arising from (4) push up the distribution when y_1 increases as compared with the naive case. On the other hand, it is enhanced when $y_1 \to 0$ by the running of $\alpha_s(k_\perp)$ at $k_\perp \to \Lambda_{QCD}$.

Transverse momentum inclusive $k_\perp$ distribution

Integrating (3) over the whole phase space in the logarithmic sense ($\ell_1 = \ln(1/x_1)$) we obtain the inclusive $k_\perp$ distribution inside a quark and a gluon jet[1]

$$\left(\frac{dN}{d\ln k_\perp}\right)_{q,g} = \int_0^{Y_{\Theta_0} - y_1} d\ell_1 \left(\frac{d^2N}{d\ell_1\, d\ln k_\perp}\right)_{q,g}. \tag{5}$$

We give results for $Y_{\Theta_0} = 7.5$ in Fig.4 below and compare with the naive case.

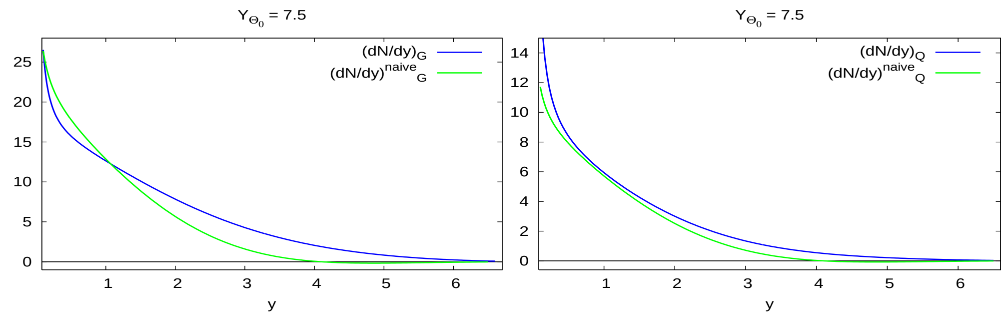

Fig. 4: $\frac{dN}{d\ln k_\perp}$ *for a gluon jet, MLLA and naive approach with enlargement*

We observe in particular that the positivity of the distribution when y_1 increases is restored as compared with the naive case. This stems from the global role of MLLA corrections in the range.

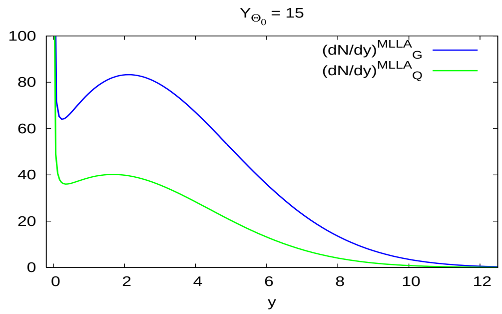

Fig. 5: $\frac{dN}{d\ln k_\perp}$ *for a gluon jet (blue) and for a quark jet (green) at* $Y_{\Theta_0} = 15.0$

In Fig.5 above, we represent the evidence of two competing effects at the unrealistic value $Y_{\Theta_0} = 15.0$. One observes that as $y_1 \to 0$ the distribution is depleted by QCD coherence effects. Indeed, in this region of the phase space ($k_\perp \to Q_0 \approx \Lambda_{QCD}$) gluons are pushed at larger angles, they are thus emitted independently from the rest of the partonic ensemble. At the opposite, when the value $k_\perp \approx Q_0 \approx \Lambda_{QCD}$ is reached, the distribution is enhanced by the running of α_s.

Conclusion

Results for the double 1-particle inclusive distribution and the inclusive $k_\perp$ distribution at small x have been discussed and displayed. Sizable differences with the naive approach in which one forgets the evolution of the jet between its half opening angle Θ_0 and the emission angle Θ have been found. The global role of new MLLA corrections is emphasized to recover the positivity of the inclusive $k_\perp$ distribution. On the other hand, at realistic energy scales (LHC, Tevatron, LEP), QCD coherence effects are screened by the running of α_s (see Fig.4) which, furthermore, forbids extending the confidence domain for $y_1 \leq 1$. The range of applicability of the soft approximation has been discussed from the analysis of corrections not to exceed $y_{max} = Y_{\Theta_0} - \ell_{min} = 5.0$; it is indeed smaller at LEP and Tevatron energies (smaller Y_{Θ_0}). Our results will be compared with forthcoming data from CDF.

References

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